Goodness of fit tests :

Example (1) A die is rolled 120 times with the following results:

	1	2	3	4	5	6
Frequency:	20	30	20	25	15	10

Let us test the hypothesis that the die is fair at level $\alpha = .05$. The null hypothesis is $H_0: p_i = \frac{1}{6}$, $i = 1, 2, \cdots, 6$, where p_i is the probability that the face value is i, $1 \le i \le 6$. By Theorem 2 we reject H_0 if

$$u = \sum_1^6 \frac{[x_i - 120(\frac{1}{6})]^2}{120(\frac{1}{6})} > \chi^2_{5,\ .05}.$$

We have

$$u = 0 + \frac{10^2}{20} + 0 + \frac{5^2}{20} + \frac{5^2}{20} + \frac{10^2}{20} = 12.5.$$

Since $\chi^2_{5,\ .05} = 11.07$, we reject H_0. Note that, if we choose $\alpha = .025$, then $\chi^2_{5,\ .025} = 12.8$, and we cannot reject at this level.

Example (2) In a 72-hour period on a long holiday weekend there was a total of 306 fatal automobile accidents. The data are as follows:

Number of Fatal Accidents per Hour	Numbers of Hours
0 or 1	4
2	10
3	15
4	12
5	12
6	6
7	5
8 or more	7

Let us test the hypothesis that the number of accidents per hour is a Poisson rv.

Since the mean of the Poisson rv is not given, we estimate it by

$$\hat{\lambda} = \bar{x} = \frac{306}{72} = 4.25.$$

$$\hat{p}_1 = .0606,\ \hat{p}_2 = .1288,\ \hat{p}_3 = .1825,\ \hat{p}_4 = .1939,$$
$$\hat{p}_5 = .1648,\ \hat{p}_6 = .1167,\ \hat{p}_7 = .0709,\ \hat{p}_8 = 1 - .9325 = .0675.$$

The observed and expected frequencies are as follows:

i:	0 or 1	2	3	4	5	6	7	8 or more
Observed frequency, 0_i:	4	10	15	12	12	6	5	7
Expected frequency $= 72\hat{p}_i = e_i$:	5.38	9.28	13.14	13.96	11.87	8.41	5.10	4.86

Thus

$$u = \sum_{i=1}^{8} \frac{(0_i - e_i)^2}{e_i} = 2.58.$$

Since we estimated one parameter, the number of degrees of freedom is $k - r - 1 = 8 - 1 - 1 = 6$. From Table 3 on page 652, $\chi^2_{6,\ .05} = 12.6$, and since $2.58 < 12.6$ we accept the null hypothesis.