

$$\begin{aligned}
E[X] &= \sum_{i=1}^{\infty} i(1-p)^{i-1}p && \text{Correction of typo from packet 1, p. 29} \\
&= p \sum_{i=0}^{\infty} i(1-p)^{i-1} && \text{Move constant } p \text{ in front and change of index.} \\
&= -p \sum_{i=0}^{\infty} \frac{d}{dp}(1-p)^i && \text{Definition of derivative of } (1-p)^i \\
&= -p \frac{d}{dp} \sum_{i=0}^{\infty} (1-p)^i && \text{Exchange of infinite sum and derivative for convergent sums} \\
&= -p \frac{d}{dp} \frac{1}{1-(1-p)} && \text{An identity that you can derive.} \\
&= p \left(\frac{1}{p^2} \right) && \text{Derivative of } \frac{1}{p} \\
&= \frac{1}{p}
\end{aligned}$$